(8 pages) **Reg. No. :** ....

Code No. : 30343 E **Sub. Code : JMMA 64/ JMMC 64**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2020.

Sixth Semester

Mathematics/ Mathematics with CA – Main

GRAPH THEORY

(For those who joined in July 2016 only)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The number of graphs a with four vertices having $\deg(v_1) = 3$ $\deg(v_2) = 2$; $\deg(v_3) = 2$; $\deg(v_4) = 2$

   (a) 1 (b) 2

   (c) 3 (d) 0

2. The maximum number of lines among all p point graphs with no triangles is __________.

(a) $\left[\frac{p^2}{4}\right]$ (b) $\left[\frac{p^2}{2}\right]$

(c) $\left[\frac{p^2}{3}\right]$ (d) $\left[\frac{p^2}{6}\right]$

3. Support T is a tree with 10 points among which 3 are pendent points then the number of cut points in T is equal to

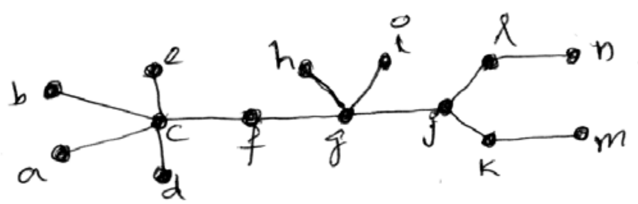
(a) 3

(b) 7

(c) any number between 1 to 7

(d) any number between 1 to 10

4. The contre(s) of the tree



(a) f, g (b) g, j

(c) h, i, g (d) g

5. If G is a maximal planar $(p,\ q)$ graph then

(a) $q = 3p + 6$ (b) $p = 3q - 6$

(c) $p = \dfrac{q+6}{3}$ (d) $q = 3p - 4$

6. If a (p, q) graph G is self dual then

(a) $q = 3p - 6$ (b) $q = 2p - 3$

(c) $q = 2p - 2$ (d) $q = 3p + 6$

7. If G is a (p, q) graph then

(a) $q \geq \binom{p}{2}$ (b) $q = \binom{p}{2}$

(c) $q < \binom{p}{2}$ (d) $q \leq \binom{p}{2}$

8. A graph G is called a line graph if

(a) $G \cong L(H)$ for some H

(b) $G \cong L(H)$ for all H

(c) $G \cong L(H)$ for one H

(d) $G \cong L(H)$ for two H

9. A wheel has chromatic number ___________ if is has an odd number of points.

(a) 4 (b) 5

(c) 3 (d) 2

10. If G is any graph then

(a) $x = \Delta + 1$ (b) $x \geq \Delta + 1$

(c) $x \leq \Delta + 1$ (d) $x = \Delta$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that every hamitonian graph is 2–connected.

Or

(b) If G is not connected then prove that $\overline{G}$ is connected.

12. (a) Prove that every connected graph has a spanning tree.

Or

(b) Prove that every non-trivial tree G has atleast two vertices of degree.

13. (a) Prove that the graph K_5 is not planar.

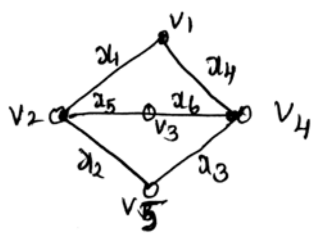
Or

(b) If G is a (p,q) plane graph in which every face is an n cycle then prove that $q = \frac{n(p-2)}{n-2}$.

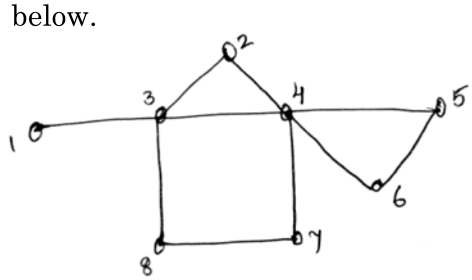
14. (a) Prove that every graph is an intersection graph.

Or

(b) Write the adjacency matrix of the graph given below.



15. (a) Write the charomatic partitioning which has chromatic number 3 of the graph given below.



Or

(b) Prove that $\lambda^4 - 3^3 + 3\lambda^2$ cannot be the chromatic polynomial of any graph.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that a graph G is connected iff for any partition of v into subsets v_1 and v_2 there is a line of G joining a point of v_1 to point of v_2.

Or

(b) Prove that a graph is hamiltonian if its closure is hamiltonian.

17. (a) Let G be a (p, q) graph. The following statements are equivalent

(i) G is a tree

(ii) Every two points of G are joined by a unique path

(iii) G is connected and $p = q + 1$

(iv) G is cyclic and $p = q + 1$.

Or

(b) Prove that every tree has a centre consisting of either one point.

18. (a) Prove that a graph can be embedded in the surface of a sphere iff it can be embedded in a plane.

Or

(b) Prove that every planar graph G with $p \geq 3$ points has atleast three points of degree less than 6.

19. (a) Let G be a (p, q) graph. Then prove that $L(G)$ is a (q, q_2) graph where $q_L = \frac{1}{2}\left(\sum_{i=1}^{p} d_i^2\right) - q$.

Or

(b) Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph. Then prove that

(i) $G_1 \cup G_2$ is a $(p_1 + p_2, q_1 + q_2)$ graph

(ii) $G_1 + G_2$ is a $(p_1 + p_2, q_1 + q_2, p_1 p_2)$ graph

(iii) $G_1 \times G_2$ is a $(p_1 p_2, q_1 p_2 + q_2 p_1)$ graph

(iv) $G_1[G_2]$ is a $(p_1 p_2, p_1 q_2 + p_2^2 q_1)$ graph.

20. (a) Prove that a matching M in a graph G is a maximum matching if and only if G contains no M-augumenting path.

Or

(b) Find the number of perfect matchings in the complete graph K_{2n}.